The m–z relation for Type Ia supernovae: safety in numbers or safely without worry?

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ABSTRACT

The m–z relation for Type Ia supernovae is compatible with the cosmological concordance model if one assumes that the Universe is homogeneous, at least with respect to light propagation. This could be due to the density along each line of sight being equal to the overall cosmological density, or to 'safety in numbers', with variation in the density along all lines of sight averaging out if the sample is large enough. Statistical correlations (or lack thereof) between redshifts, residuals (differences between the observed distance moduli and those calculated from the best-fitting cosmological model), and observational uncertainties suggest that the former scenario is the better description, so that one can use the traditional formula for the luminosity distance safely without worry.

Key words: supernovae: general – cosmological parameters – cosmology: observations – cosmology: theory – dark energy – dark matter.

1 INTRODUCTION

I recently investigated the dependence of constraints on the cosmological parameters λ_0 and Ω_0 derived from the m–z relation for Type Ia supernovae on the degree of local homogeneity of the Universe (Helbig 2015). When deriving such constraints, it is often assumed that the Universe is completely homogeneous, at least with regard to light propagation. However, the constraints on the cosmological parameters derived depend on this assumption. If the degree of local inhomgeneity is parametrized by the parameter η giving the fraction of homogeneously distributed matter on the scale of the beam size such that the density at a given redshift is equal to the average cosmological density $\rho = \frac{3H^2\Omega}{8\pi G}$ outside the beam and $\eta\rho$ inside the beam (see Kayser, Helbig & Schramm 1997, for definitions and discussion), and assuming that η is independent of redshift and the same for all lines of sight, then only if $\eta \approx 1$ do the constraints on the cosmological parameters λ_0 and Ω_0 derived from the m–z relation for Type Ia supernova correspond to the 'concordance model' (e.g. Ostriker & Steinhardt 1995; Komatsu et al. 2011; *Planck* Collaboration 2014).

Two important conclusions of Helbig (2015) are thus that the values of λ_0 and Ω_0 derived from the m–z relation for Type Ia supernovae depend on assumptions made about η, substantially so for current data, and that only for $\eta \approx 1$ are these values consistent with other measurements of

the cosmological parameters. Perlmutter et al. (1999) considered the effect of $\eta \neq 1$ on their results (see their fig. 8 and the discussion in their section 4.3) and concluded that, at least in the 'interesting' region of the λ_0–Ω_0 parameter space (i.e. $\Omega_0 < 1$; even at that time there was substantial evidence against $\Omega_0 > 1$), it had a negligible effect. Not only is this effect no longer negligible with newer data (both because there are more data points altogether and because there are more data points at higher redshifts), but, especially since we now have good estimates of λ_0 and Ω_0 from other tests, it allows one to use the supernova data to say something about η. With a strong indication from the supernova data that $\eta \approx 1$, it is important to consider the question whether this is true only when several lines of sight are averaged or is true for a typical individual line of sight. Perlmutter et al. (1999) investigated the influence of η on the values obtained for λ_0 and Ω_0 but could draw no conclusions about its value from the supernova data alone. Even though the 'concordance model' had already been postulated at the time (though of course there was much less evidence in favour of it than is the case today), assuming the corresponding values for λ_0 and Ω_0 could not allow any statement to be made about η since there was significant overlap in the allowed regions of parameter space for the various η scenarios. (Also, in contrast to the case with newer data, the best-fitting values of λ_0 and Ω_0 were far from the concordance values, though the concordance values were allowed even at $1\,\sigma$.) This is consistent with their claim, based on simulations, that the conclusions drawn from their data should not depend heavily on η. Their robust conclusion that the m–z relation for Type Ia

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supernovae implies that $\lambda_0 > 0$, and the somewhat stronger claim that $q_0 < 0$ (i.e. the Universe is currently accelerating), regardless of assumptions made about η, are of course the most interesting results of Perlmutter et al. (1999) (and similar studies by the High–z Supernova Search Team and later papers by both groups). Interestingly, both of these are still robust with current data.

2   TWO SCENARIOS

There are two ways in which $\eta \approx 1$ can be explained. One is that $\eta \approx 1$ holds for each individual line of sight. (This does not necessarily imply that η is actually constant along the beam, but only that the distance modulus calculated from the cosmological parameters λ_0, Ω_0, and H_0 and from the redshift z is the same as that calculated assuming $\eta \approx 1$. In other words, there could be density variations along the beam (apart from the decrease in density with decreasing redshift due to the expansion of the Universe) as long as they appropriately average out.) The other is that $\eta < 1$ for some lines of sight and $\eta > 1$ for others, such that $\eta \approx 1$ when averaged over all lines of sight, though of course density variations along the beam as in the other case could also be present.[1] This has been dubbed the 'safety in numbers' effect by Holz & Linder (2005).

In the first case, the residuals (the differences between the observed distance moduli and those calculated from the best-fitting cosmological parameters) should not depend on redshift per se, while in the second case they should increase with redshift: all else being equal, the lower η, the larger the distance modulus, and the difference between this and that calculated using the traditional $\eta = 1$ assumption is a monotonically increasing function of redshift; see e.g. fig. 1 in Kayser et al. (1997).[2] In the first case, the residuals are due only to uncertainties in the observed distance moduli, while in the second case they are due also to variations in the actual distance moduli as a result of different average densities along the line of sight. Of course, in the first case there could be a dependence of the residuals on redshift if the observational uncertainties depend on redshift, and in the second case the residuals are due both to variations in the actual distance moduli and to observational uncertainties in them. One could call the first case 'safely without worry', meaning

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[1] The second case requires a more general definition of η than that used in Kayser et al. (1997); see Lima, Busti & Santos (2014) and Helbig (2015) for discussion. Strictly speaking, as pointed out by Weinberg (1976), it is the magnification μ which averages to 1 over all lines of sight. Since $\eta \sim \kappa$, where κ is the convergence, and $\mu \sim (1 - \kappa)^{-1}$, the relation is linear only in the limit of vanishing deviations, though approximately linear for the small deviations considered here. The actual situation is quite complicated. For example, the average angular-size distance $\langle D \rangle$, and hence the average luminosity distance $\langle D_L \rangle$, is biased even in the case of $\langle \mu \rangle = 1$. See Kaiser & Peacock (2015) for discussion of this and many other details in the still ongoing debate on this topic. I use the term 'average' here loosely; the important point is that the average of an observed quantity is the same as in the $\eta = 1$ case, not that η itself averages to 1.

[2] There is of course a similar effect with opposite sign for $\eta > 1$, as discussed in the previous footnote.
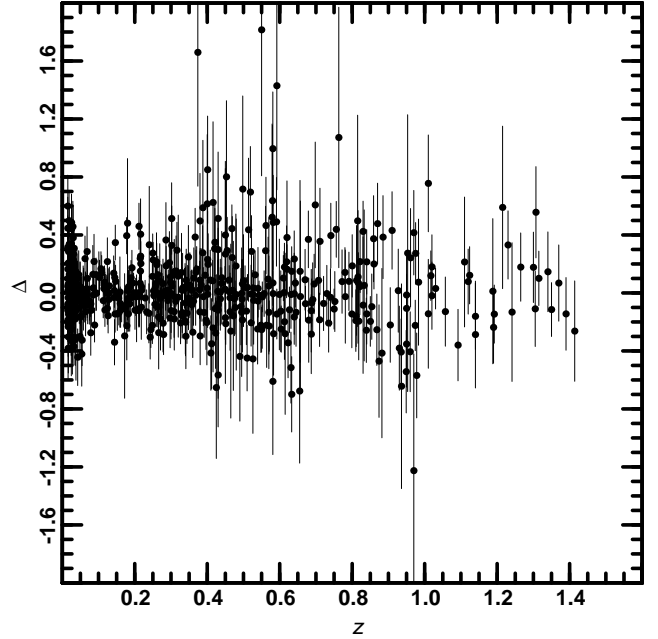

Figure 1. Residuals (differences between the observed distance moduli and those calculated from the best-fitting cosmological model) (points) and observational uncertainties (lines).

that one can safely use the traditional formula for the luminosity distance (corresponding to $\eta = 1$) when calculating the distance modulus, without worry.

3   CALCULATIONS, RESULTS, AND DISCUSSION

For purposes of comparison and consistency, I work with the same data as in Helbig (2015), namely the publicly available 'Union2.1' sample of supernova data (Suzuki et al. 2012). Fig. 1 shows the residuals Δ (points) with respect to the best-fitting model assuming $\eta = 1$ in Helbig (2015) ($\lambda_0 = 0.7210938$ and $\Omega_0 = 0.2773438$) and the uncertainties σ in the distance moduli (lines). These are shown separately (both as points) in Figs. 2 and 3. There appear to be a positive correlation between the uncertainties and redshifts and a higher number of outliers at intermediate redshifts (though the fact that there are fewer at high redshifts might be due to the smaller number of objects there). If the first case discussed above holds, then (the absolute value of) the quotient $Q = \Delta/\sigma$ of the residuals and the uncertainties should show no trend with redshift, while if the second case holds there should be a positive correlation. Fig. 4 shows this quotient and, indeed, there appears to be no trend with redshift. Also, the width of the distribution seems to depend only on the number of points in the corresponding redshift range, i.e. there appear to be no outliers as such, or at least fewer.

In order to quantify the dependence of the magnitude of the uncertainties on redshift, I have calculated various statistical measures, shown in Table 1, to investigate the existence of a correlation between the redshifts and the absolute values of the residuals $|\Delta|$ (plotted in Fig. 5), the observational uncertainties σ (Fig. 3), and the absolute value of the quotient of the residuals and the uncertainties, $|Q|$ (plotted in
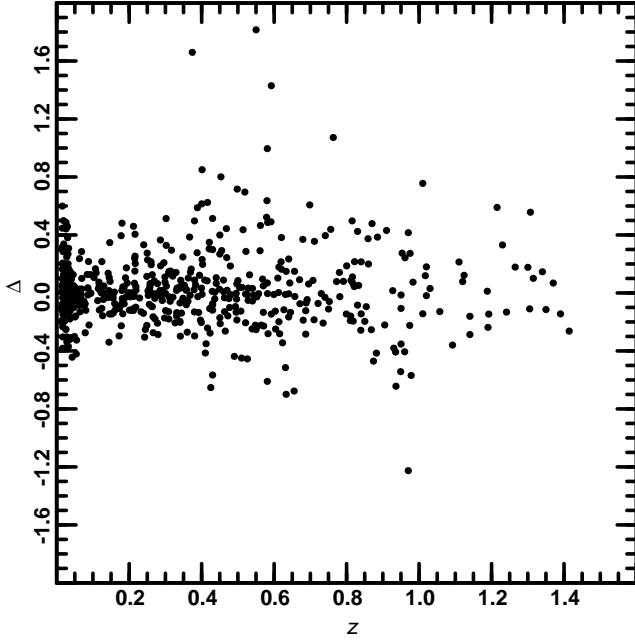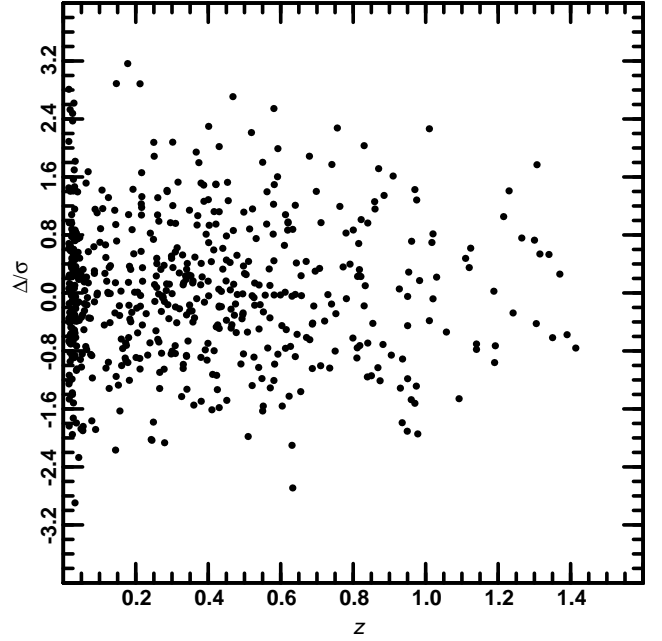
Figure 2. Residuals.

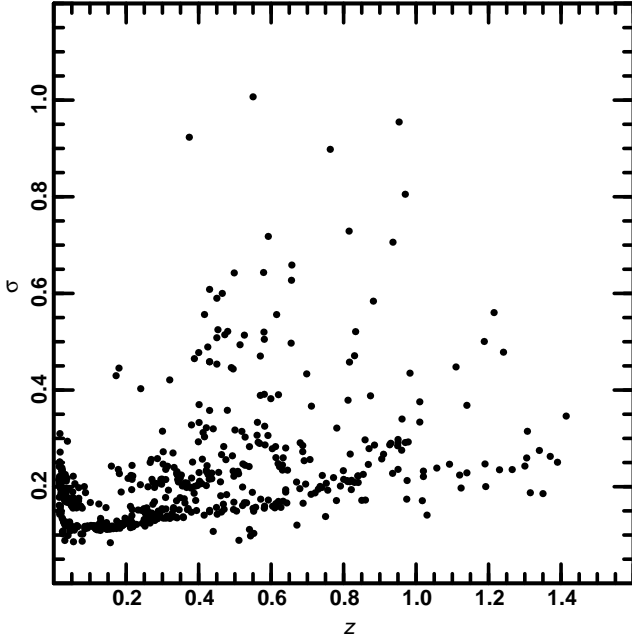**Figure 4.** Quotients of residuals and observational uncertainties.

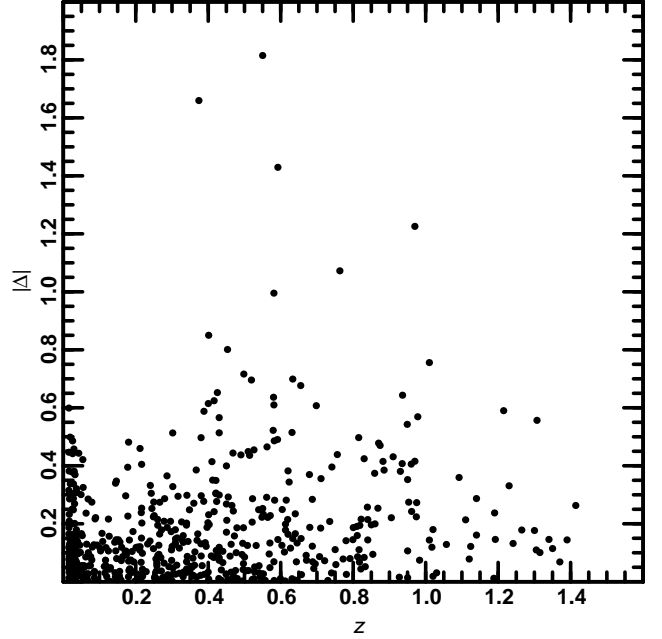**Figure 3.** Observational uncertainties.

Figure 5. Absolute values of residuals.

Fig. 6), as well as the corresponding statistical significance. Note that Fig. 5, like Fig. 3, appears to show a positive correlation between the absolute values of the residuals and redshifts and a higher number of outliers at intermediate redshifts. Also included in Table 1 are the corresponding quantities concerning the correlation between $|\Delta|$ and σ.

All three statistical tests agree about the sign of the correlation and whether or not it is significant. (The *values* of the correlations and the corresponding significance are not directly comparable.) Both the absolute values of the residuals, $|\Delta|$, and the observational uncertainties, σ, are

positively correlated with redshift, but their quotient is not. This suggests that the first scenario described above, 'safely without worry', is the appropriate one, not the second scenario, 'safety in numbers'. If this is the case, then one would expect $|\Delta|$ and σ to be correlated, and indeed they are. (Note that this last test is not sufficient to rule out the 'safety in numbers' scenario, since even if the scatter in the actual distance moduli increased with redshift, there could still be a correlation between $|\Delta|$ and σ in addition to the one between $|\Delta|$ and z.)

Of course, this analysis takes the Union2.1 data set at

Table 1. Statistical quantities measuring the correlation between the redshifts z and the absolute values of the residuals $|\Delta|$, the observational uncertainties σ, and the quotient $|Q|$ of these, as well as between $|\Delta|$ and σ: r is Pearson's product-moment correlation coefficient, r_s is Spearman's rank-order correlation coefficient, and τ is Kendall's non-parametric rank-order correlation coefficient. The corresponding p values give the probability of getting a value as large as observed or larger in the case of the null hypothesis of no correlation. All values have been rounded to two significant figures.

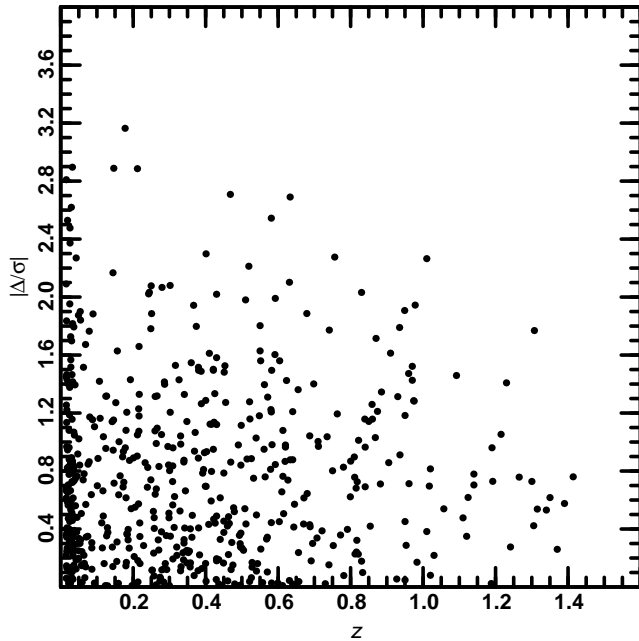| data set	r	$p(r)$	r_s	$p(r_\mathrm{s})$	τ	$p(\tau)$
$z,|\Delta|$	0.23	1.3×10^{-8}	0.21	1.8×10^{-7}	0.14	7.8×10^{-7}
z,σ	0.41	1.5×10^{-25}	0.44	4.5×10^{-29}	0.28	2.6×10^{-23}
$z,|Q|$	3.6×10^{-2}	0.39	5.1×10^{-2}	0.22	3.3×10^{-2}	0.24
$|\Delta|,\sigma$	0.62	0.00	0.42	1.3×10^{-25}	0.29	1.7×10^{-25}

Figure 6. Quotients of absolute values of residuals and observational uncertainties.

face value, and relies on the assumption that the observational uncertainties have been correctly estimated. Also, $\eta \neq 1$ describes just a different amount of Ricci focusing due to more or less matter within the beam than in the standard case, as opposed to more general gravitational lensing. Note that Suzuki et al. (2012) explicitly correct for the amplification of supernovae known to be gravitationally lensed by galaxy clusters (see their section 2.1); in other words, the magnitudes used for the cosmology analysis are those which would have been observed in the absence of the corresponding galaxy clusters. To be sure, Suzuki et al. (2012), following the procedure described in section 7.3.5 of Amanullah et al. (2010), include as part of the error estimate $0.093z$ to take the statistical uncertainty due to gravitational lensing into account. If this were a significant part of the uncertainty, then it could explain the correlation between the uncertainties and redshifts, and thus favour the 'safety in numbers' scenario. This contribution to the error budget probably explains the slope of the lower envelope in Fig. 3. However, it is clear from Figs. 3 and 4 that the main cause of the correlation is the absence of both large residuals and large uncertainties at low redshifts. The large residuals – much larger than $0.093z$ – also have large uncertainties, and

occur mainly at intermediate redshifts. Finally, as described in section 7.2 of Amanullah et al. (2010) and section 4.4 of Suzuki et al. (2012), the Union2.1 data set was constructed by rejecting 3-σ outliers, which would remove any strongly lensed supernovae from the sample. Both this use of median statistics and the $0.093z$ contribution contribute to the correlation at some level but, as explained above, cannot explain all, or even most, of it.

Note that Yu et al. (2011), using observational data other than the m–z relation for Type Ia supernovae, and assuming a flat Universe, arrive at essentially the same conclusion as Helbig (2015): $\eta \approx 1$ is favoured and low values of η can be ruled out.[3] Some of the assumptions in Yu et al. (2011) were questioned by Busti & Santos (2011), but even when these are corrected for, $\eta \approx 1$ is still favoured. As discussed in Helbig (2015), one expects to measure a larger value of η when larger angular scales, such as those investigated in Yu et al. (2011), are considered, so the result of Helbig (2015) remains interesting because of the small angular scales of supernova beams.

The *Planck* Collaboration (2015) measured the CMB lensing-deflection power spectrum at $40\,\sigma$, showing it to agree with the smooth ΛCDM amplitude (i.e. the $\eta = 1$ case) to within 2.5 per cent. Since all forms of gravitating clumps contribute to this, such a measurement of the power as a function of scale is fairly definitive about the smoothness of the energy-density distribution. This should be contrasted with the situation a few decades ago, when it was widely believed that there was no dark matter other than that required for flat rotation curves in spiral galaxies and for bound galaxy clusters; $\eta \approx 0$ was thought to be the best approximation even for objects as large as large galaxies (e.g. Gott et al. 1974; Roeder 1975). To be sure, most of the analysis done by the *Planck* Collaboration (2015) deals with $L \leq 400$, although $L < 2048$ is also investigated, where L is the multipole. $L = 400$ corresponds to an angular scale of somewhat less than a degree and $L = 2048$ to about 10 arcmin. This means that the corresponding physical size in the concordance model is about 5 Mpc at $z = 1$ (and about 40 kpc at the redshift of the CMB). Thus, the m–z relation for Type Ia supernovae probes much smaller scales, and indicates that even at these scales $\eta \approx 1$ is appropriate, i.e. that the Universe is homogeneous at even these very small scales.

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[3] Yu et al. (2011) usually refer to (Ruth) Daly et al. (2008) as 'Ruth et al.'.

4 CONCLUSIONS

There is a statistically significant correlation between the absolute value of the residuals, i.e. the difference between the observed distance moduli and those calculated from the best-fitting cosmological model, and the observational uncertainties in the Union2.1 sample of Type Ia supernova observations. Each of these quantities is also correlated with redshift but their quotients are not. This suggests that each individual line of sight to these supernovae is a fair sample of the Universe in the sense that the (average) density is approximately the same as the overall density; in other words, it is not necessary to average over several lines of sight in order to recover the overall density. Since most of the matter in the Universe is dark matter, it must be distributed smoothly enough so that most lines of sight contain the same density as the overall average density. When the resolution of cosmological numerical simulations becomes high enough to resolve the corresponding scale, this distribution must result. Rather than putting it in 'by hand', it would be more interesting if it emerged from other assumptions or theoretical considerations.

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